As Geoscientists, we are accustomed to facing uncertainty, and thus often provide not only a Best Estimate, but also both Up-and Down-side. In fact, however, while this simple and useful characterization is a step in the right direction, it can be improved upon in a manner that recognizes (Figure 1);

- It is unlikely (but not impossible) that the various input High- and Low-Side values will occur simultaneously.

- The individual input attributes (Rw, Porosity, etc in Archie’s equation, for example) are linked, and a change in the uncertainty of one can affect the impact that another has on the ultimate estimate.

- Determines which of the input attributes is dominating uncertainty in the ultimate estimate, for each specific combination.

There are two basic alternatives to the High- and Low-Side approach, partial derivatives and statistical simulation, that complement one another.

Here the issue is illustrated with Archie’s equation, but the concept is general, in that once understood it may be applied to many of the issues we face day-to-day (routine and special core analyses, conversion of Pc(Lab) to Pc(Reservoir), Saturation(Height), Reservoir Volumetrics, etc).

Once we have mastered one calculation, it is straightforward to apply it to an entirely different question. And the Excel spreadsheet that we have constructed for the first calculation, jump-starts the evaluation of the next calculation.
The advantage of Monte Carlo simulation is that once the spreadsheet is set up for a specific model, it is straight-forward to modify it for a completely different question. Additionally, non-Gaussian distributions, which do indeed occur in the oilfield, can be addressed and the visual distribution output facilitates an extra dimension of understanding: Figure 2.

The attraction of the differential approach is that it is a set of analytical equations result, which may be easily coded into a foot-by-foot evaluation, and then displayed alongside our best-estimate results: Figure 3.

Remember, in the Figure 3 symbolism (following Chen & Fang, 1986) that uncertainty for each attribute is written simply as y%, but the specific value is in general unique to each parameter and situation. Note that the uncertainty in Sw, associated with Porosity, ‘m’ and ‘n’ is linked to other parameters and uncertainty estimates.

The Differential Approach

As carbonate (rather than shaly sand) petrophysicists, our Sw estimates are typically compromised by uncertainty in the Archie equation attributes.

\[ S_w^n = a \frac{R_w}{(\Phi^m R_t)} \]

By taking the derivative of Archie’s equation (the same approach will suffice for a shaly sand equation, or any of the other various calculations Geoscientists routinely perform), one is able to quantify the individual impact of each term (and its uncertainty) upon the ultimate result, and thus recognize where the biggest bang for the buck, in terms of a core analyses program, suite of potential logs, etc, is to be found.

Each of the Sw input attributes has a Best Estimate value, and an associated uncertainty distribution. The individual uncertainties ‘propagate’ through to the composite result according to a specific protocol: http://en.wikipedia.org/wiki/Propagation_of_uncertainty.

The uncertainty of each parameter ‘x’ may be characterized by the respective standard deviation (\( \sigma_x \)), which is the positive square root of the variance (\( (\sigma_x)^2 \)). As an example, the 68% confidence limits of a normally (bell-shaped) distributed variable ‘x’, are \( x \pm \sigma_x \).

In general, the uncertainty in ‘y’, which is some function of variables \( x_1 \rightarrow x_n \), is

\[ (\sigma_y)^2 = \sum [(dy/\partial x_i) \sigma x_i]^2 \]

where the partial derivative of y, with respect to \( x_i \), is represented by \( dy/\partial x_i \) rather than the mathematically correct partial derivative symbol, so as to minimize the need for special word
processor symbols. See the Wikipedia reference, above, for more background information and the partial derivative symbolism; there are additional mathematical details in Appendix V of the Mechanics Lab Manual, Case Western Reserve University (available on-line): Uncertainty and Error Propagation.

As a specific example of error propagation, Ohm’s Law relates Resistance (R), Voltage (V) and Current (I) as

\[ R = \frac{V}{I} \]

Measurement of both “V” and “I” are subject to uncertainty, which propagates through to “R”, according to (as above)

\[ \Delta R^2 = (\Delta V / I)^2 + (V \Delta I / I^2)^2 \]

Differentiation of Archie’s equation is somewhat more complicated than Ohm’s Law, but still within the realm of basic calculus (Risky Business for the details). Following Chen (1986) the respective uncertainties are thought of as percentage wise specifications (“y%” in Chen’s notation, in general unique for each attribute, but uniformly symbolized as y% in the notation).

\[ \frac{dS_w}{dR_w} = \frac{S_w}{(n \ast R_w)} \]

\[ (dS_w / dR_w)^2 = (S_w / n)^2 [(\sigma R_w) / R_w]^2 = (S_w / n)^2 [(\gamma \% R_w) / R_w]^2 = (S_w / n)^2 (\gamma \%)^2 \]

\[ \frac{dS_w}{da} = \frac{S_w}{(n \ast a)} \]

\[ (dS_w / da)^2 = (S_w / n)^2 [(\sigma a) / a]^2 = (S_w / n)^2 [(\gamma \% a) / a]^2 = (S_w / n)^2 (\gamma \%)^2 \]

\[ \frac{dS_w}{dR_t} = \frac{S_w}{(n \ast R_t)} \]

\[ (dS_w / dR_t)^2 = (S_w / n)^2 [(\sigma R_t) / R_t]^2 = (S_w / n)^2 [(\gamma \% R_t) / R_t]^2 = (S_w / n)^2 (\gamma \%)^2 \]

\[ \frac{dS_w}{d\phi} = -m \frac{S_w}{(n \phi)} \]

\[ (dS_w / d\phi)^2 = (S_w / n)^2 [m (\sigma \phi) / \phi]^2 = (S_w / n)^2 [m (\gamma \% \phi) / \phi]^2 = (S_w / n)^2 (m \gamma \%)^2 \]

\[ \frac{dS_w}{dm} = \frac{S_w}{(n \ast n)} \ln(\phi)(-1/n) \]

\[ (dS_w / dm)^2 = (S_w / n)^2 [\ln(\phi) \ln(\sigma m)]^2 = (S_w / n)^2 [\ln(\phi)(\gamma \% m)]^2 = (S_w / n)^2 [m \ln(\phi) \gamma \%]^2 \]

\[ \frac{dS_w}{dS_w} = \frac{S_w}{(n \ast n)} \ln(S_w)(-1/n) \]

\[ (dS_w / dS_w)^2 = (S_w / n)^2 [\ln(S_w) \ln(\sigma n)]^2 = (S_w / n)^2 [\ln(S_w)(\gamma \% n)]^2 = (S_w / n)^2 [n \ln(S_w) \gamma \%]^2 \]

Each of the terms share the \((S_w / n)^2\) prefix; dropping this yields the relative magnitudes of the various components, as follows.

\[ C(R_w) = (\gamma \%)^2 \]
\[ C(a) = (\gamma \%)^2 \]
\[ C(R_t) = (\gamma \%)^2 \]
\[ C(\phi) = (m \gamma \%)^2 \]
C(m) = [m ln(\phi) y\%]^2
C(n) = [n ln(S_w) y\%]^2

For complete clarity and comparison, we follow Chen’s 1986 nomenclature, as in Figure 3.

These equations reveal that ‘\(a\)’, \(R_w\) and \(R_t\) are dependent upon only their respective uncertainties, whereas \(\phi\), ‘m’ and ‘n’ involve other attributes.

When the respective uncertainties are equal, one then has \(C(\phi) > C(R_w)\), \(C(a)\), \(C(R_t)\) simply because ‘m’ > 1: Figure 4. In such a situation, time and money are better spent on improved porosity estimates, rather than ‘a’, \(R_w\) or \(R_t\).

Because the relative uncertainty in ‘m’ and ‘n’ involves the (square of a) natural logarithm, the behavior is more complicated, and involves an inflection point.

A spreadsheet formulation allows one to not only easily perform the calculations, but to also consider what the effect of a change in reservoir quality (porosity) would mean (because the importance of ‘m’ and ‘n’ is linked to porosity), with locally specific values.

At 20 pu formation evaluation, with Chen’s conditions, the focus should be on improved porosity and ‘m’ estimates, with ‘n’ of relatively less importance. If porosity rises to 30 pu, however, improved porosity estimates become more important with ‘m’ and ‘n’ having similar, and less, impact. As porosity drops below 20 pu, it is the pore connectivity (‘m’) that begins to dominate the accuracy: Figure 5.
If the water were fresher, say Rw = 0.2 instead of 0.02, ‘n’ diminishes in importance as compared to both the amount of porosity, and its connectivity (‘m’).

Yet more details and variations are to be found in Chen & Fang (1986); coding the equations to a spreadsheet will allow application of the concept to locally specific conditions. And an advantage of the differential approach is that it may be readily accommodated within our foot-by-foot evaluations.

In this, or other physical models, we must remember that not only is there uncertainty in the various inputs to the model (equation), but there may very well be limitations upon the validity of the equation being used: Figure 6.

Carlos Torres-Verdin cautions “my experience shows that the biasing of apparent resistivity curves due to post-processing techniques (e.g. deconvolution) could be more detrimental to uncertainty than Archie's parameters. A conspicuous case is that of a thin, hydrocarbon-saturated bed: bed thickness and invasion can give you much more uncertainty than Archie's parameters”.

Monte Carlo Simulation

Monte Carlo simulation is based upon repeated random sampling of the various user-specified input distributions, which are then used to calculate a composite result. Multiple ‘random calculations’ super-imposed and a statistical representation of the ‘expected distribution’ results. It’s a particularly attractive approach when it is infeasible or impossible to compute an exact result with a deterministic algorithm, and we also find the visual display of the simulated distribution to be useful.

In the case of the illustrative Sw simulation, each of the individual input attributes (Rw, Phi, ‘m’, etc) can be represented by the appropriate Best Estimate value, encompassed by a distribution of associated Possible Values, according to the appropriate user-specified distribution (Gaussian, Square, Triangular, Log Normal, etc).

The individual, randomly selected results for Rw, Phi, ‘m’, etc are then input to the Sw calculation, multiple times, and the resulting Most Likely Value, and uncertainty distribution, determined both numerically and graphically.

A limitation of Monte Carlo is that special software is often used (commercial add-ons to Excel, etc), and may not even an option in commercially available petrophysics s/w packages.
Common oilfield distributions, however, such as Normal, Log Normal and Triangle are available in Excel and it is straight-forward to implement Monte Carlo within the Excel framework. In this approach, one remains in the familiar Excel environment, and actually leverages their Excel skill set via the additional hands-on experience within the platform.

A discussion of the Monte Carlo method can be found in Decision Analysis for Petroleum Exploration by Paul Newendorp & John Schuyler, and a collection of articles addressing exploration risk can be found in The Business of Petroleum Exploration published by the AAPG, Tulsa, Oklahoma.

Additional information may be found in the References, with useful on-line reference material to be found at the following links.

- http://www.enrg.lsu.edu/pttc/
- http://www.mrexcel.com/
- http://people.stfx.ca/bliengme/exceltips.htm
- http://en.wikipedia.org/wiki/Monte_carlo_simulation
- http://www.sitmo.com/eqcat/15

For illustration purposes (and as was done with the differential example), we regard “a”, Rw and Rt to be well-known, and Φ, “m” and “n” subject to uncertainty as specified in Figure 7.

Allowance for uncertainty in “a”, Rw and Rt may be addressed by a straight-forward extension of the techniques presented here. Also, while the focus here is on the simple Sw(Archie), any other model (shaly sand, core analyses, capillary pressure, etc) may be evaluated in a similar manner. Once the concepts are understood, locally specific models are readily developed.

Figure 7

*Each of the uncertain attributes are modeled as a random number input to NormInv, whose mean value and standard deviation are locally appropriate. For example, the first pass random estimate of porosity, with a distribution centered on 20 pu and having a standard deviation of 1 pu, results in an estimate of 21 pu.

• As a quality control device, we determine and display the distribution of random numbers, between zero and one, for the number of Monte Carlo passes being used in a specific simulation (2000, in this example). In a perfect world there would then be 200 observations in each of the ten bins displayed.

• As a quality control device, we determine and display the distribution of random numbers, between zero and one, for the number of Monte Carlo passes being used in a specific simulation (2000, in this example). In a perfect world there would then be 200 observations in each of the ten bins displayed.
This example is based upon Gaussian distributions, but any of the other Excel options (Log Normal, for example) that also occur in the oilfield, may be substituted.

*Each of the uncertain attributes is modeled as a random number input to NormInv, whose mean value and standard deviations are locally appropriate* (and specified by the User). For example (Exhibit 7), the first pass random estimate of porosity, with a distribution centered on 20 pu and having a standard deviation of 1 pu, results in an estimate of 21 pu. The random values of “m” and “n”, appropriate to the specified distributions, are independently and randomly determined, and $S_w$ calculated per the Archie relation.

Because Excel recalculates equations each time the spreadsheet is opened, or specifications are changed, the various results will change (your line item spreadsheet values will change, each time you make a modification).

As a *quality control device*, we determine and display the distribution of random numbers, between zero and one, for the number of Monte Carlo passes being used in a specific simulation (2000, in this example). *In a perfect world there would be 200 observations in each of the ten blue bins displayed in Figure 7.*

We typically set up the spreadsheet with all input values specified in a single worksheet, and links of those values to other relevant worksheets for display. Then, in order to prevent an accidental over-write, we protect the cells for which the displayed values are the result of a link. As an example, the specifications reported in the upper left of Figure 7 are ‘linked values’, to allow easy comparison of the specification and the individual, multiple random realizations (below the Spec Table). By protecting the cells in the Spec Table, we avoid an accidental, inappropriate over-write.

![Figure 8](image)

**Figure 8** illustrates the relation between the random magnitude of NormInv, and the distribution of NormInv values, for different standard deviations, at 90 simulations. Both distributions take on an approximate Gaussian appearance, with the larger standard deviation result displaying more scatter. As the number of random calculations is increased, each distribution will approach the ideal Gaussian. *It is the distribution of NormInv values that is driving the Sw(Archie) simulation.*

*simulation*. It’s important to realize that each occurrence of NormInv involves an independent Rand() input.
The **approach taken here is intended to parallel that of the LSU results** (Must Read supplemental material), which also includes Log Normal and Triangle distributions, and so can be directly referenced if either of those distributions are required: www.enrg.lsu.edu/pttc/.

**As an additional QC device**, the statistical attributes of the simulated quantities (Φ, 'm' and ‘n’ in this example) are tabulated **directly from the simulation population**, and displayed graphically: Figure 9.

With 2000 simulations (easily handled by Excel), the model population nicely replicates the input numerical specifications, and the porosity distribution takes on the expected appearance per the parameter specs.

Simulation results are reported both numerically and graphically: Figure 10. In this particular case, there is a **95% likelihood that Sw is contained within + / - 2 σ** (0.357 – 0.076) < Sw < (0.357 + 0.076) \( \Rightarrow \) 0.28 < Sw < 0.43. The graphical display adds another dimension to the calculation.

In utilizing Excel frequency distribution graphics, one should **take note of how the ‘bins’ are populated, as they are not ‘centered’**. This can cause the graphic to take on a shifted appearance, with respect to the numerical report (consult Excel Help on the Frequency function for details).
In addition to the Monte Carlo simulation, we also set the spreadsheet is up to calculate the Best / Worst case scenario, which is found to significantly over-state the 95% Monte Carlo uncertainty. It is unlikely (though not impossible) that the Best or Worst, of all attributes, would occur simultaneously: Figure 11.

A contrast of Figure 10 vs Figure 11 illustrates the ‘randomness’ of the simulation, in that the spreadsheet was closed after generating Figure 10, and reopened to produce Figure 11. As a result, we see slight differences in the various calculated (but not specified) numerical values. As the number of random simulations is increased beyond 2000, this difference diminishes, and vice versa.

*The Biggest Bang for the Buck would be determined by* now varying the individual input specs, one after another, and observing which realistically achievable incremental improvement results in the greatest improvement in Sw.

In viewing the graphics and results, one must bear in mind that the Sw(Archie) result population is affected by the nonlinear relation between the various attributes, as discussed by Bryant et al in Understanding Uncertainty, Oilfield Review. Autumn 2002, who illustrate that a normal uncertainty distribution about a given porosity yields a log-normal distribution for the resulting Sw distribution. *Bryant’s article is another Must Read.*

**Dealing With Risk**

With rigorous mathematical options in hand, we are now able to quantitatively estimate the uncertainty in an ultimate estimate, once we characterize the individual input parameter uncertainties. This parameter specification is itself, a challenge, and subject the qualification of ‘one size does not fit all feet’.


**Summary points** include

- A single interpreter should avoid making estimates on their own.
• A single interpreter often lacks the needed knowledge to correctly estimate every parameter.
• In addition, many interpreters have a bias that smaller errors are better and they will appear more knowledgeable about the subject.
• The error must reflect the level of knowledge about the parameters and the data quality.
• **A standard set of uncertainty ranges must be avoided** because there is no standard situation in which to apply them.
• **Unusual events also pose special problems.**
  • Most people have a better recall of unusual events
    • Therefore a tendency to overestimate the probability of such an event
    • Especially if that event occurred recently
• Another very common mistake is to allow a very small amount of data to quantify the range of uncertainty
  • If data sets are small, the ranges probably need to be increased.
• Boundary Conditions
  • Water saturation must lie between zero and one
  • If the saturation values are too large or too small, the "best guesses” and ranges must be reconsidered and calculations remade.
• **The final and probably most difficult problem to overcome is the culture and preconceived ideas of an organization.**
  • **Methods and ranges of uncertainty applied to any analysis must be questioned every time they are applied.**

In the case of the illustrative $S_w$ analyses, we should **not lose sight of the fact** that the porosity estimate is the result of a transform, which itself involves uncertainty (remember that old phrase; the devil is in the details). Likewise, the $S_w$ model involves basic assumptions. For example, if clay or conductive minerals are an issue, the $S_w$ (Archie) evaluation has been immediately compromised.

In the carbonate world, as compared to clastic, clay is much less commonly a problem but dual porosity systems, particularly in the transition zone, can challenge the Archie equation electrical circuit model: Griffiths et al (2006), Figure 6. High in the oil column or for gas wells in general, increased capillary pressure will cause the hydrocarbon to more efficiently displace the brine, and the magnitude of the problem diminishes.

**Acknowledgements**

This one is for colleagues who have attended my Carbonate Petrophysics course, and whose questions and comments have given me so many new perspectives and ideas.
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**Biography**

R. E. (Gene) Ballay’s 35 years in petrophysics include research and operations assignments in Houston (Shell Research), Texas; Anchorage (ARCO), Alaska; Dallas (Arco Research), Texas; Jakarta (Huffco), Indonesia; Bakersfield (ARCO), California; and Dhahran, Saudi Arabia. His carbonate experience ranges from individual Niagaran reefs in Michigan to the Lisburne in Alaska to Ghawar, Saudi Arabia (the largest oilfield in the world).

He holds a PhD in Theoretical Physics with double minors in Electrical Engineering & Mathematics, has taught physics in two universities, mentored Nationals in Indonesia and Saudi Arabia, published numerous technical articles and been designated co-inventor on both American and European patents.

At retirement from the Saudi Arabian Oil Company he was the senior technical petrophysicist in the Reservoir Description Division and had represented petrophysics in three multi-discipline teams bringing on-line three (one clastic, two carbonate) multi-billion barrel increments. Subsequent to retirement from Saudi Aramco he established Robert E Ballay LLC, which provides physics - petrophysics consulting services.

He served in the U.S. Army as a Microwave Repairman and in the U.S. Navy as an Electronics Technician, and he is a USPA Parachutist and a PADI Dive Master.